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5, Differentiate the following with respect to x

$$\frac{e^{4x}\sin x}{x\cos 2x}$$

Sol

Let $y = \frac{e^{4x}\sin x}{x\cos 2x}$

Let $u = e^{4x}\sin x$

from u, let $t = e^{4x}$, $\frac{dt}{dx} = 4e^{4x}$

$S = \sin x$, $\frac{ds}{dx} = \cos x$

$$\frac{du}{dx} = S\frac{dt}{dx} + t\frac{ds}{dx}$$

$$\frac{du}{dx} = \sin x(4e^{4x}) + e^{4x}(\cos x)$$

$$= 4e^{4x}\sin x + e^{4x}\cos x$$

$$\frac{du}{dx} = e^{4x}(4\sin x + \cos x)$$

Let $V = x\cos 2x$

from V, Let $P = x$, $\frac{dP}{dx} = 1$

$w = \cos 2x$, $\frac{dw}{dx} = -2\sin 2x$

$$\frac{dv}{dx} = w\frac{dP}{dx} + P\frac{dw}{dx}$$

$$= \cos 2x(1) + x(-2\sin 2x)$$

$$\frac{dv}{dx} = \cos 2x + -2x\sin 2x$$

$$\frac{dy}{dx} = \frac{V\frac{du}{dx} - u\frac{dv}{dx}}{V^2}$$

$$\frac{dy}{dx} = \frac{x\cos 2x\left[e^{4x}(4\sin x + \cos x)\right] - e^{4x}\sin x(\cos 2x - 2x\sin 2x)}{(x\cos 2x)^2}$$

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$$\frac{dy}{dx} = \frac{x\cos 2x\,(4e^{4x}\sin x + \cos x) - e^{4x}\sin x\,(\cos 2x - 2x\sin 2x)}{x^2\cos^2 4x^2}$$

$$\frac{dy}{dx} = \frac{4xe^{4x}\sin x\cos 2x + x\cos^2 2x^2 - e^{4x}\sin x\cos 2x + 2xe^{4x}\sin 2x}{x^2\cos^2 4x^2}$$

$$\frac{dy}{dx} = \frac{e^{4x}\sin x\cos 2x\,(4x-1) + x\cos^2 2x^2 + 2xe^{4x}\sin 2x}{x^2\cos^2 4x^2}$$

$$dy/dx = \frac{e^{4x}\sin x\cos 2x\,(4x-1) + x\,(\cos^2 2x^2 + xe^{4x}\sin 2x)}{x^2\cos^2 4x^2}$$

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6. $\frac{x^4}{(x+1)^2}$

Let $u = x^4$, $du/dx = 4x^3$

$V = (x+1)^2$ $dv/dx = 2\cdot(x+1)^{2-1}\cdot 1 = 2(x+1) = 2x+2$

$$dy/dx = \frac{V\frac{du}{dx} - u\frac{dv}{dx}}{V^2}$$

$$= \frac{(x+1)^2(4x^3) - x^4(2x+2)}{((x+1)^2)^2}$$

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$$= \frac{4x^3(x+1)^2 - x^4(2x+2)}{(x+1)^4}$$

$$= \frac{4x^3(x^2+2x+1) - 2x^5 - 2x^4}{(x+1)^4}$$

$$= \frac{4x^5 + 8x^3 + 4x^3 - 2x^5 - 2x^4}{(x+1)^4}$$

$$= \frac{4x^5 - 2x^5 - 2x^4 + 12x^3}{(x+1)^4}$$

$$= \frac{2x^5 - 2x^4 + 12x^3}{(x+1)^4} \quad \text{OR} \quad \frac{2x^5 - 2x^4 + 12x^3}{3x^4 + 2x^3 + 6x^2 + 4x + 1}$$

6. $y = \ln(3 - 4\cos x)$

Sol

$y = \ln(3 - 4\cos x)$

Let $u = 3 - 4\cos x$

$\cancel{\frac{dy}{dx} = \frac{1}{3-4\cos x}}$

$y = \ln u$

$\frac{dy}{dx} = \frac{1}{u} \cdot u'$

$\frac{dy}{dx} = \frac{1}{3-4\cos x} \cdot (3-4\cos x)'$

$dy/dx = \frac{4\sin x}{3-4\cos x}$

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2. If $x^2 + y^3 - 4x + 4y = 26$ find dy/dx and $\frac{d^2y}{dx^2}$

Sol

$x^2 + y^3 - 4x + 4y = 26$

$2x + 3y^2 \frac{dy}{dx} - 4 + 4\frac{dy}{dx} = 0$

$dy/dx\ 3y^2 + 4 = 4 - 2x$

$\frac{dy}{dx} = \frac{4-2x}{3y^2+4}$

To get d^2y/dx^2

Let $u = 4 - 2x$

$v = 3y^2 + 4$

$\frac{du}{dx} = -2$

$\frac{dv}{dx} = 6y\frac{dy}{dx}$

$\frac{d^2y}{dy^2} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$= \frac{(3y^2+4)(-2) - (4-2x)(6y\frac{dy}{dx})}{(3y^2+4)^2}$

$= \frac{(-6y^2-8) - (24y\frac{dy}{dx} - 12xy\frac{dy}{dx})}{(3y^2+4)^2}$

$= \frac{(-6y^2-8) - 24y\frac{dy}{dx} + 12xy\frac{dy}{dx}}{(3y^2+4)^2}$

$\cancel{= -6y^2 - 8 - 24y + 12xy}$

$= \frac{-6y^2 - 8 - (24y - 12xy)\frac{dy}{dx}}{(3y^2+4)^2}$

$= \frac{-6y^2 - 8 - (24y - 12xy)\left[\frac{4-2x}{3y^2+4}\right]}{(3y^2+4)^2}$

$= \frac{-6y^2 - 8 - \left[\frac{96y - 48yx}{3y^2+4} - \frac{48xy - 24x^2y}{3y^2+4}\right]}{(3y^2+4)^2}$

$\frac{-6y^2 - 8 - \left[\frac{96y - 48yx - 48xy + 24x^2y}{3y^2+4}\right]}{(3y^2+4)^2}$

$$\frac{d^2y}{dx^2} = \frac{\frac{-6y^2 - 8 - 96y + 48yx + 48xy - 24x^2y}{3y^2+4}}{(3y^2+4)^2}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{3y^2+4(-6y^2) - 8(3y^2+4) - 96y + 48yx + 48xy - 24x^2y}{3y^2+4}}{(3y^2+4)^2}$$

$$\frac{d^2y}{dx^2} = \frac{3y^2+4(-6y^2-8) - 96y + 96xy - 24x^2y}{3y^2+4} \times \frac{(3y^2+4)^2}{1}$$

$$= (3y^2+4)^2(-6y^2-8) - (96y + 96xy - 24x^2y)(3y^2+4)$$

$$\frac{d^2y}{dx^2} = 3y^2+4\left[3y^2+4(-6y^2-8) - 96y - 96xy + 24x^2y\right]$$

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